Integral calculus

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Definition 1. Let $f(x)$ be a function, and let $f'(x)$ be its derivative. The reverse process of differentiation is called *antidifferentiation* or *integration*. It gives us the original function, which is called the *antiderivative* or *integral* of $f(x)$.

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Theorem 1. Let c, n and k be constants. Then

a.
$$\int k dx = kx + c$$

b.
$$\int dx = x + c$$

c.
$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + c, \quad n \neq -1$$

d.
$$\int x^{-1} \, dx = \ln x + c, \quad x > 0$$

e.
$$\int x^{-1} \, dx = \ln |x| + c, \quad 0 \neq x < 0$$

f.
$$\int e^{kx} \, dx = \frac{1}{k} e^{kx} + c$$

g.
$$\int kf(x) dx = k \int f(x) dx$$

h.
$$\int (f(x) \pm g(x)) = \int f(x) dx \pm \int g(x) dx$$

i.
$$\int -f(x) dx = - \int f(x) dx$$

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Definition 2. The approximation $\sum_{i=1}^{n} (f(x_i) \, \Delta x^i)$ of the area under a continuous curve A is called a *Riemann sum*. That area under the curve is

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x_i$$

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Theorem 2. Let $F(x)$ be the integral of $f(x)$. We call the *fundamental theorem of calculus* the expression.

$$\int_a^b f(x) dx = F(x)|_a^b = F(b) - F(a)$$

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Theorem 3.

a.
$$\int_a^b f(x) dx = - \int_b^a f(x) dx$$

b.
$$\int_a^a f(x) dx = F(a) - F(a) = 0$$

c.

$$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx, \quad a \le b \le c$$

d.

$$\int_a^b f(x)dx \pm \int_a^b g(x)dx = \int_a^b (f(x) \pm g(x))dx$$

e.

$$\int_a^b kf(x)dx = k\int_a^b f(x)dx$$

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Property d of Theorem 3 is used to find the area between two curves.

Theorem 4. The process of *integration by parts* is

$$\int (f(x) \cdot g'(x))dx = f(x) \cdot g(x) - \int (g(x) \cdot f'(x))dx$$

Proof. From

$$\frac{d}{dx}(f(x) \cdot g(x)) = f(x) \cdot g'(x) + g(x) \cdot f'(x)$$

we have

$$f(x) \cdot g(x) = \int (f(x) \cdot g'(x))dx + \int (g(x) \cdot f'(x))dx$$

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Bibliography

Edward T Dowling. *Mathematical methods for business and economics.* Schaum's outline series, 1993